NuMI-L-612
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Physics potential of the three Ph2 beam designs

Abstract

The purpose of this note is to compare the ability of the three Ph2 beam designs to detect and measure neutrino oscillations in the region of parameter space suggested by the Super-Kamiokande atmospheric neutrino analysis. These results are intended to guide the choice of beam for initial MINOS physics runs.

1 Introduction

The predicted sensitivity of MINOS to neutrino oscillations has been explored in a number of NuMI notes. Several of the earlier notes are of somewhat limited usefulness at this time, due to the changing nature of the experiment. In addition, it is sometimes difficult to obtain a clear picture of the overall merits of the three currently proposed beam designs from the various analyses. The purpose of this note is therefore to summarise, in a handful of plots, the sensitivity to oscillations of each of the three beams using a range of analysis methods.

The oscillation tests studied in this note can be broken down into three categories:

- **Primary hypothesis tests:** The purpose of these tests is to establish the existence of neutrino oscillations by measuring a statistically significant difference in the nature of the neutrinos measured in the near and far detectors. The two tests studied here are the T-test (which measures the NC/CC ratio) and the Z-test (the statistic of which is derived from the measured energies of ν_μ CC events). The T-test in particular is a simple, statistically powerful test that is relatively immune to systematic errors. The ν_μ CC disappearance test also falls into this category and detailed studies of this test can be found in references [1, 2].

- **Primary parameter measurement methods:** These tests are performed once the hypothesis tests have yielded evidence for oscillations and their purpose is to provide a measurement of the mixing parameters. A summary of the results of the ν_μ CC energy spectrum analysis is presented here. More detailed studies can be found in references [1, 2, 3].

- **Appearance tests:** These tests search for events that are consistent with ν_e CC or ν_τ CC interactions and can therefore provide information on the oscillation mode. Such measurements are complementary to the tests listed above and at least one appearance test is required to constrain the oscillation parameters in a three-flavour framework. A summary of ν_e appearance signals is presented here and the prospects of ν_τ appearance are discussed.

For sake of brevity, the details of these analysis are not presented here. References to NuMI notes in which such information can be found are included in the text. The efficiencies and resolutions used throughout are based on studies performed using the GEANT-based GMINOS Monte Carlo system.

These results should be taken as a guide only. Future advances in analysis techniques and better understanding of the systematic errors may modify the results, and hence change the conclusions.

2 Primary hypothesis tests

2.1 The T-test

The T-test measures the ratio of the number of identified ν_μ CC to the total number of events in both the near and far detectors. A statistically significant difference between these two ratios is evidence of neutrino oscillations. While the value of the ratio at a particular value of Δm^2 and $\sin^2 2\theta$ is weakly sensitive to the oscillation mode, the T-test cannot, on its own, either measure the parameters nor determine the oscillation mode. It is purely a hypothesis test; the hypothesis being oscillations versus no oscillations.

The algorithm that is used to discriminate between ν_μ CC and NC events is described in [4]. Two sets of cuts are employed: a simple event length cut (at 35 planes) which is effective for neutrinos with energies greater than 2 GeV is used for the Ph2he beam; and a sequence of cuts which is efficient for neutrinos of lower energy and uses the Hough transform, is used for the two lower energy beams. The efficiency of these cuts for ν_μ CC events is shown in figure 1, along with the ν_μ CC interaction spectra for the three beams under consideration. These efficiencies, which assume a light yield equivalent to 1.6 times the "September '97 yield", are used throughout this document.[1]

NuMI-L-481 presented the 90% exclusion limits that could be set using the T-test for each of the three beams. This is the standard way of presenting the sensitivity of an experiment to oscillations and can be used to compare the relative merits of the three Ph2 beams. Given the evidence for oscillations provided by Super-Kamiokande however, it is perhaps more useful to consider what would be observed by MINOS if the following assumptions are adopted: $\nu_\mu \to \nu_\tau$ oscillations with $\sin^2 2\theta = 1$ and $10^{-3} < \Delta m^2 < 10^{-2}$ eV^2, with the most probable value of Δm^2 being the Super-K best-fit point: 0.0035 eV^2 [5]. Figure 2 shows the number of standard deviations that would be observed in the T-test for these assumptions, the three Ph2 beams and a 10 kiloton year exposure of MINOS. The shaded area represents the range of Δm^2 favoured by Super-K at 90% C.L. and the dashed line shows the Super-K best fit value of Δm^2.

There are three separate plots in figure 2. The top plot assumes only statistical errors, while the other two plots incorporate possible errors on the ν_μ CC selection efficiency and the neutral current trigger efficiency. Uncertainties on these quantities are the most important components of the systematic error on T [4, 6] and are parameterised in the following way in this analysis:

[1] This is roughly equivalent to the light yields reported in the TDR.

- an overall energy-independent uncertainty in the CC efficiency, α:

$$\eta(E_\nu) = \eta(E_\nu) \times (1 + \alpha \times r),$$

where $\eta(E_\nu)$ is the ν_μ CC selection efficiency and r is a Gaussian-distributed random number;

- an additional energy-dependent uncertainty in the CC efficiency below $E_\nu = 4$ GeV, characterised by β:

$$\eta(E_\nu) = \eta(E_\nu) \times (1 + \frac{1}{2}(4 - E_\nu) \times \beta \times r).$$

This ensures that the uncertainty in the efficiency is greatest at and below the 'shoulder' in $\eta(E_\nu)$ (see figure 1). The rationale behind this functional form comes from figure 8 of [4], which shows the effect of light yield on the functional form of $\eta(E_\nu)$;

- an overall energy-independent uncertainty in the NC trigger efficiency (a simple 4/5 plane trigger is used), characterised by γ:

$$\epsilon(E_\nu) = \epsilon(E_\nu) \times (1 + \gamma \times r),$$

where ϵ is the NC trigger efficiency.

The systematic errors on T for several values of these parameters are listed in table 1. The numbers in parentheses are for Ph2he, where the simplified CC selection algorithm is considered to be less subject to systematic error than the Hough transform algorithm employed for Ph2le and Ph2me.

Error source	**Ph2le**	**Ph2me**	**Ph2he**
Statistical	3.6×10^{-3}	2.0×10^{-3}	1.2×10^{-3}
$\alpha = 1\%, \beta = 2\%(1\%), \gamma = 1\%$	5.6×10^{-3}	3.8×10^{-4}	3.1×10^{-3}
$\alpha = 2\%, \beta = 4\%(2\%), \gamma = 2\%$	1.1×10^{-2}	7.5×10^{-3}	6.2×10^{-3}

Table 1: Comparison between statistical and systematic errors in the T test. The statistical errors assume a 10 kiloton year exposure of MINOS.

2.2 The Z-test

The Z-test, which is described in detail in [7], is a technique for detecting neutrino oscillations from the measured energies of ν_μ CC events. The quantity Z for a particular hypothesised value of Δm^2 is defined as follows:

$$Z = 1/N_{ev} \sum_{events} \cos(2.54 \Delta m^2 L/E_i),$$

where N_{ev} is the number of events in the sample and E_i is the measured energy of the i-th event. The quantity Z is calculated as a function of Δm^2 for events in both near and far detectors: $Z_n(\Delta m^2)$ and $Z_f(\Delta m^2)$. A significant difference between these two functions is evidence of oscillations. In addition, the peak position and half-width of the function $\Delta Z(\Delta m^2)(= Z_f(\Delta m^2) - Z_n(\Delta m^2))$ provides a measurement of Δm^2. Figure 3 shows examples of $\Delta Z(\Delta m^2)$ distributions for simulated experiments with maximal two-fold neutrino oscillations and several values of Δm^2 between 0.002 and 0.01 eV^2. Curves for each of the three beam designs are plotted and perfect energy resolution is assumed.

The statistical significance of a signal in the Z-test is defined as follows:

$$\sigma_Z = \Delta Z_{max}/\sqrt{var(Z)},$$

with

$$var(Z) = 1/2N_f(1 - 2Z^2(\Delta m_m^2) + Z(2\Delta m_m^2),$$

where N_f is the number of identified ν_μ CC events in the far detector and Δm_m^2 is the value of Δm^2 that maximises $\Delta Z(\Delta m^2)$. The top plot of figure 4 shows the value of σ_Z plotted as a function of Δm^2 for the three beams, assuming statistical errors only.[2] The bottom plot shows the fractional difference between the 'measured' value of $\Delta m^2 : \Delta m_m^2$ and the true value of $\Delta m^2 : \Delta m_0^2$, for each of the fits (37 fits generated with Δm_0^2 between 0.001 and 0.01 eV^2 in steps of 0.00025 eV^2 are performed for each beam). This plot shows that, for this method, significant biases in the measured value of Δm^2 are possible if $\Delta m^2 << \bar{E}/L$, where $\bar{E}$ is the mean interaction energy of the beam in question and L is 735 km - the baseline of the experiment.[3]

The two plots in figure 5 show how systematic errors affect the statistical significance of the Z-test signal. The top plot assumes that the error in predicting the far detector spectrum from the near spectrum is $\pm 2\%$ in each 1 GeV energy bin and that there is a $\pm 2\%$ uncertainty in the energy calibration between the two detectors. The bottom plot assumes a $\pm 4\%$ uncertainty for each of these systematic errors. The additional contribution to $var(Z)$ that arises from these errors is calculated as follows: a no oscillation far detector data sample is compared to a near detector data sample with the relevant systematic effect applied. The maximum value of $\Delta Z(\Delta m^2)$ calculated from these two samples between $0 < \Delta m^2 < 0.025$ eV^2 is then assumed to be the systematic component of $var(Z)$. Table 2 lists the square roots of the components of $var(Z)$ for each of the systematic effects and the three beams. Systematic effects have more of an impact on sensitivity in the higher energy beams; in the low energy beam the statistical error is quite large and is comparable to the systematic errors considered here.

3 Primary parameter measurement methods

3.1 ν_μ CC energy test

The procedure adopted here follows that of [1] and [3]. The aim of this analysis is to fit the reconstructed energy distributions of oscillated ν_μ CC events in order to determine

[2]These, and subsequent plots assume the following energy resolutions: $\Delta p_\mu/p_\mu = 10\%$ and $\Delta E_h/E_h = 60\%/\sqrt{E}$.

[3]Oscillations at low Δm^2 can be fit over a large range of Δm^2 and $\sin^2 2\theta$. This method appears to favour the larger values of Δm^2. Section 3 describes an alternative way to measure Δm^2 and $\sin^2 2\theta$ which does not possess this feature.

Error source	Ph2le	Ph2me	Ph2he
Statistical	1.5×10^{-2}	8.4×10^{-3}	5.1×10^{-3}
2% energy calibration	1.4×10^{-2}	2×10^{-2}	2.1×10^{-2}
4% energy calibration	3×10^{-2}	3.7×10^{-2}	4.2×10^{-2}
2% bin-to-bin	1.4×10^{-2}	5×10^{-3}	4×10^{-3}
4% bin-to-bin	1.4×10^{-2}	5×10^{-3}	7×10^{-3}

Table 2: Comparison between statistical and systematic errors in the Z test.

the errors on the mixing parameters: Δm^2 and $\sin^2 2\theta$. The events are generated with the energy resolutions and CC selection efficiencies described previously. Figure 6 shows a comparison between the unoscillated energy distributions in the three beams and oscillated energy distributions with $\Delta m^2 = 0.003$ eV^2 and $\sin^2 2\theta = 0.8$. The unoscillated distribution is then weighted by a pair of oscillation parameters chosen from a grid of $\Delta m^2, \sin^2 2\theta$ values and a χ^2 between the two distributions is calculated. The definition of χ^2 follows that of [1], although possible systematic errors associated with NC contamination of the CC sample are ignored. The assumed systematic errors are as follows: 4% overall flux normalisation error, 4% uncorrelated bin-to-bin flux error and 4% overall CC selection efficiency uncertainty[4]

Figure 7 shows the 68% C.L. contours in parameter space ($\chi^2_{min} + 2.3$ for 2 d.o.f.) that are obtained from fits to oscillated distributions in each of the three beams. The four plots show the results of separate fits to the following values of Δm^2 : $0.002, 0.003, 0.005$ and 0.1 eV^2, all with $\sin^2 2\theta = 0.8$.

These plots are summarised in figure 8, which shows the fractional error on the measurement of the oscillation parameters (i.e. the size of the 68% C.L. contours) for the three beams as a function of Δm^2. A precise measurement of the parameters is desirable for a number of reasons; from allowing powerful cross-checks of the results of various MINOS analyses (for example an accurate measurement of Δm^2 allows a prediction of the number of ν_τ events that could be observed in MINOS), to setting bounds on quantities, such as CP violating amplitudes, that could be measured by future experiments. For the purpose of this discussion, a 'good' measurement of the parameters is defined as $\Delta(\Delta m^2)/\Delta m^2$ and $\Delta(\sin^2 2\theta)/\sin^2 2\theta \leq 10\%$.

4 Appearance tests

4.1 Electron appearance

NuMI-L-576 recently provided an updated analysis of the potential of MINOS to detect $\nu_\mu \to \nu_e$ oscillations [8]. While the results of Super-Kamiokande and CHOOZ indicate that the level of $\nu_\mu \to \nu_e$ at the atmospheric neutrino scale must be small ($< 10\%$), it is nevertheless of interest to attempt to detect evidence of oscillations below this level. The

[4] a 2% error is assumed for Ph2he, given that the CC selection algorithm (a simple event length cut) is much simpler than the Hough transform method that is adopted for Ph2le and Ph2me.

amount of $\nu_\mu \to \nu_e$ present at the atmospheric neutrino scale is crucial to understanding the nature of the three-flavour mixing matrix and may be one of the few qualitatively unique measurements that MINOS can make when it begins data taking in the year 2003.

The analysis here follows closely that of NuMI-L-576. A series of cuts are applied that are efficient in selecting ν_e CC events and rejecting the neutral current background. The numbers of identified ν_e CC events in the near and far detectors are compared and a χ^2 is formed between the two. Figure 9 shows the significance of the expected ν_e signal for two mixing hypotheses. The top plot shows the number of standard deviations expected for two-fold $\nu_\mu \to \nu_e$ mixing with $\sin^2 2\theta = 1$ and $10^{-3} < \Delta m^2 < 10^{-2}$ eV^2. The bottom plot shows the χ^2 between the numbers of electron-like events in near and far detectors assuming a three-flavour mixing scenario with large $\nu_\mu \to \nu_\tau$ mixing and subdominant $\nu_\mu \to \nu_e$ mixing constrained by the results of the CHOOZ experiment [8, 9].

4.2 Tau appearance

Tau appearance in the MINOS far detector has not been studied in depth for some time. It is clear that high energies and event rates are desirable to detect tau appearance signals, due to the tau production threshold ($\sim$ 3.5 GeV). In addition, analyses that search for exclusive tau modes, such as $\tau \to e$ and $\tau \to \pi$ have acceptances for ν_τ CC events that are of the order of 1% in order to reduce the level of background contamination [10, 11]. Figure 10 shows the tau production rates expected for $\nu_\mu \to \nu_\tau$ oscillations with $\sin^2 2\theta = 1$ and $10^{-3} < \Delta m^2 < 10^{-2}$ eV^2, assuming a 10 kiloton year exposure in each of the three beams. At the Super-Kamiokande best fit point, approximately 200 taus are produced in the high energy beam. An analysis with an acceptance of 1% will yield two signal events at this value of Δm^2. It is therefore clear that exclusive tau analyses in MINOS are difficult in the Super-K region, regardless of the choice of beam. It may be possible to observe a 3σ or greater signal in Ph2he (and maybe Ph2me) if Δm^2 is close to 10^{-2} eV^2.

An inclusive tau signal can be obtained by examining the hit distributions of neutral current events. This does not incur the acceptance penalties that affect the exclusive mode analyses and may extent the $\nu_\mu \to \nu_\tau$ reach to lower values of Δm^2. Such a signal is important for three flavour analyses and, along with electron identification signals, allows the three-flavour matrix elements to be constrained or measured.

5 Summary and conclusions

5.1 Oscillation physics in the year 2003

In order to differentiate between the three beams, we must first consider what the likely experimental situation will be in the year 2003. The current best indicator of the oscillation parameters is provided by the Super-Kamiokande results, and this is still likely to be the case in three years time. However, while the oscillation signal in Super-Kamiokande is relatively robust, further developments in calculations of the atmospheric neutrino flux may well change the allowed region in parameter space [12].

The K2K experiment may provide some additional information; it might not be able to provide a precise measurement of the parameters, but the observation of a large ($\sim 50\%$) suppression of the ν_μ flux will tend to disfavour the lowest values of Δm^2 currently allowed by Super-Kamiokande. In addition, it is expected that the cross-section for π^0 production will be measured in the K2K near detector, allowing Super-Kamiokande to discriminate between $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_{sterile}$ oscillations with atmospheric neutrino data. If the $\nu_\mu \to \nu_\tau$ oscillation hypothesis is convincingly demonstrated in Super-Kamiokande then there could be a change in emphasis of future MINOS appearance measurements. Observing τ leptons in MINOS would then become merely a check of the Super-K results, whereas observing $\nu_\mu \to \nu_e$ oscillations at the atmospheric neutrino scale would be a qualitatively new, and arguably more interesting measurement.

5.2 MINOS measurements for a 10 kiloton year run

In this discussion it is assumed that the initial physics run of MINOS will be a 10 kiloton year exposure (approximately 2 calendar years, assuming 100% livetime). At the end of this run, we clearly want to be able to demonstrate the existence of a statistically significant oscillation signal in a number of independent and complementary measurements. For definiteness, I define the following three 'essential' requirements for the initial physics run:

- A 5 standard deviation signal or greater in the T-test;
- A 5 standard deviation signal or greater in the Z-test;
- Measurement of the mixing parameters to 10% accuracy or better.

and the following important, but non-essential, requirements:

- Measure a difference between the expected and observed number of electron-like events at 99% C.L., assuming the CHOOZ limit on U_{e3}^2 [8];
- Produce at least 100 ν_τ CC interactions in the far detector; this should be enough to observe a τ signal in an emulsion detector or perhaps produce a measurable effect in the number and hit distribution of neutral current events in the main detector.

Figure 11 shows, for the three beams, the range of Δm^2 for which each of these requirements are satisfied[5]. The following trends are clearly apparent from this figure, and from the preceding plots in this note:

- The low energy beam, Ph2le, is superior in the lower reaches of the Super-Kamiokande allowed region, namely $0.002 < \Delta m^2 < 0.0035$ eV^2.
- The 'cross-over point' between Ph2le and Ph2me is somewhere between 0.0035 and 0.005 eV^2 depending on the particular test and assumed systematic errors.
- The higher energy beams (Ph2me and Ph2he) are superior for appearance tests, except for a small range of $\Delta m^2 \sim 0.003$ eV^2 where the low energy beam provides the largest (although in itself quite modest) signal for electron appearance.
- The 'essential' requirements are met over almost the entire Super-Kamiokande allowed region for Ph2le and for $\Delta m^2 > 0.0035$ eV^2 for Ph2me.

[5] Where there is more than one estimate of the systematic error, as in the T-test and the Z-test, the smaller error is chosen here.

5.3 Conclusions

From these results it is clear that if we wish to maximise our chances of observing an oscillation signal over the entire Super-K allowed region then Ph2le is the beam of choice, mainly because it offers increased sensitivity over the other beams at low values of Δm^2. However, the low energy beam has limited flavour identification capabilities compared to Ph2me and Ph2he. A possible running strategy therefore, is to first measure the oscillation parameters with Ph2le and then switch to Ph2me or Ph2he to observe any appearance signals.[6] The one exception to this rule is if the Super-K/K2K results in 2003 indicate that $\Delta m^2 > 0.005$ eV^2, in which case Ph2me is the best choice for initial running to observe both disappearance and appearance signals.

References

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[3] D. A. Petyt, *"A Study of Parameter Measurement in a Long-baseline Neutrino Oscillation Experiment'*, D. Phil. thesis, University of Oxford, February 1998.

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[5] Y. Fukuda *et. al.*, The Super-Kamiokande Collaboration, Phys. Lett. **B436**, p.33.

[6] The MINOS Collaboration, *"P-875: A Long-baseline Neutrino Oscillation Experiment at Fermilab*, Fermilab Proposal P-875, Fermilab internal note, NuMI-L-63, February 1995.

[7] J. H. Cobb, *"A Test for Neutrino Oscillations using the Energy Distributions of ν_μ Charged Current Events"*, Fermilab internal note, NuMI-L-106, August 1995.

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[9] M. Apollonio *et. al.*, The CHOOZ collaboration, hep-ph/9907037.

[10] The MINOS Collaboration, *"Status Report on Tau Identification in MINOS"*, Fermilab internal note, NuMI-L-228, January 1997.

[11] D. A. Petyt, *"$\tau \rightarrow \pi + X$ analysis in MINOS"*, Fermilab internal note, NuMI-L-258, March 1997.

[12] P. Lipari, hep-ph/0002282.

[6] This assumes that the value of Δm^2 measured by the Ph2le run indicates that ν_e or ν_τ appearance signals will be observable in Ph2me or Ph2he

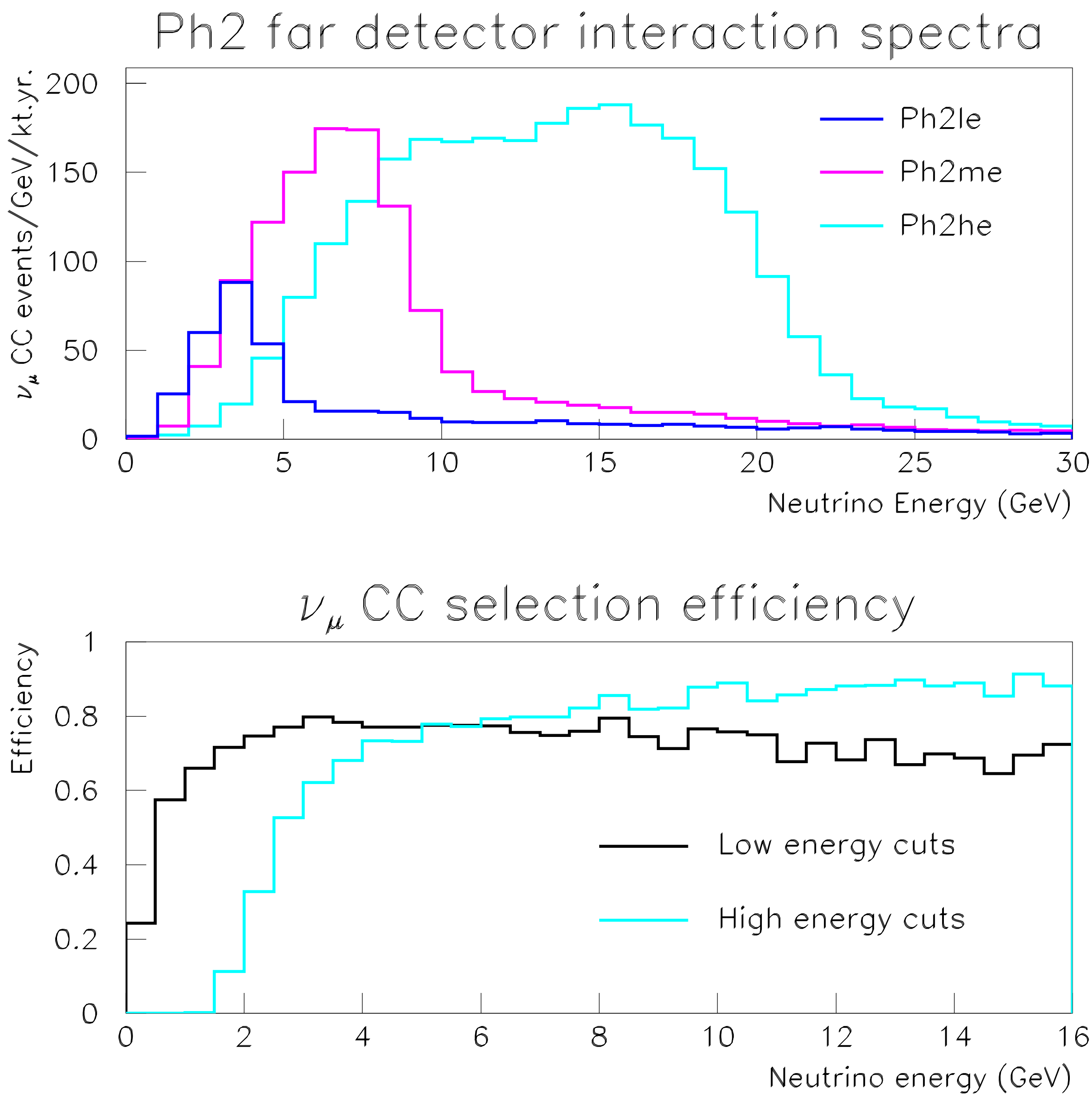


Figure 1: Top plot: the ν_μ CC interaction spectra in the far detector for each of the three beams. Bottom plot: the ν_μ CC selection efficiencies that are used in this document.

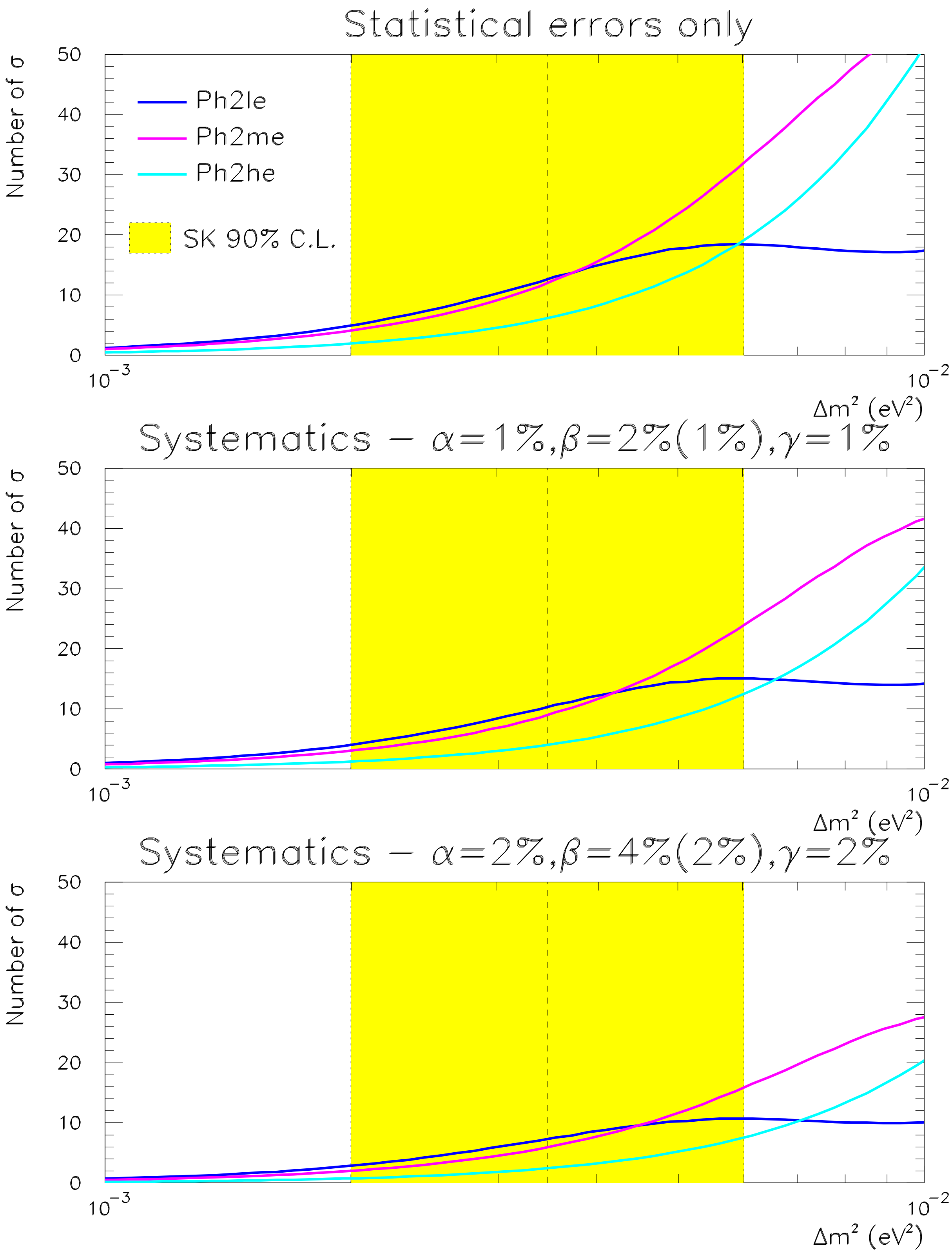


Figure 2: Statistical significance of the T-test signal for the three beams. $\nu_\mu \to \nu_\tau$ oscillations with $\sin^2 2\theta = 1$ and a 10 kiloton year exposure of MINOS are assumed.

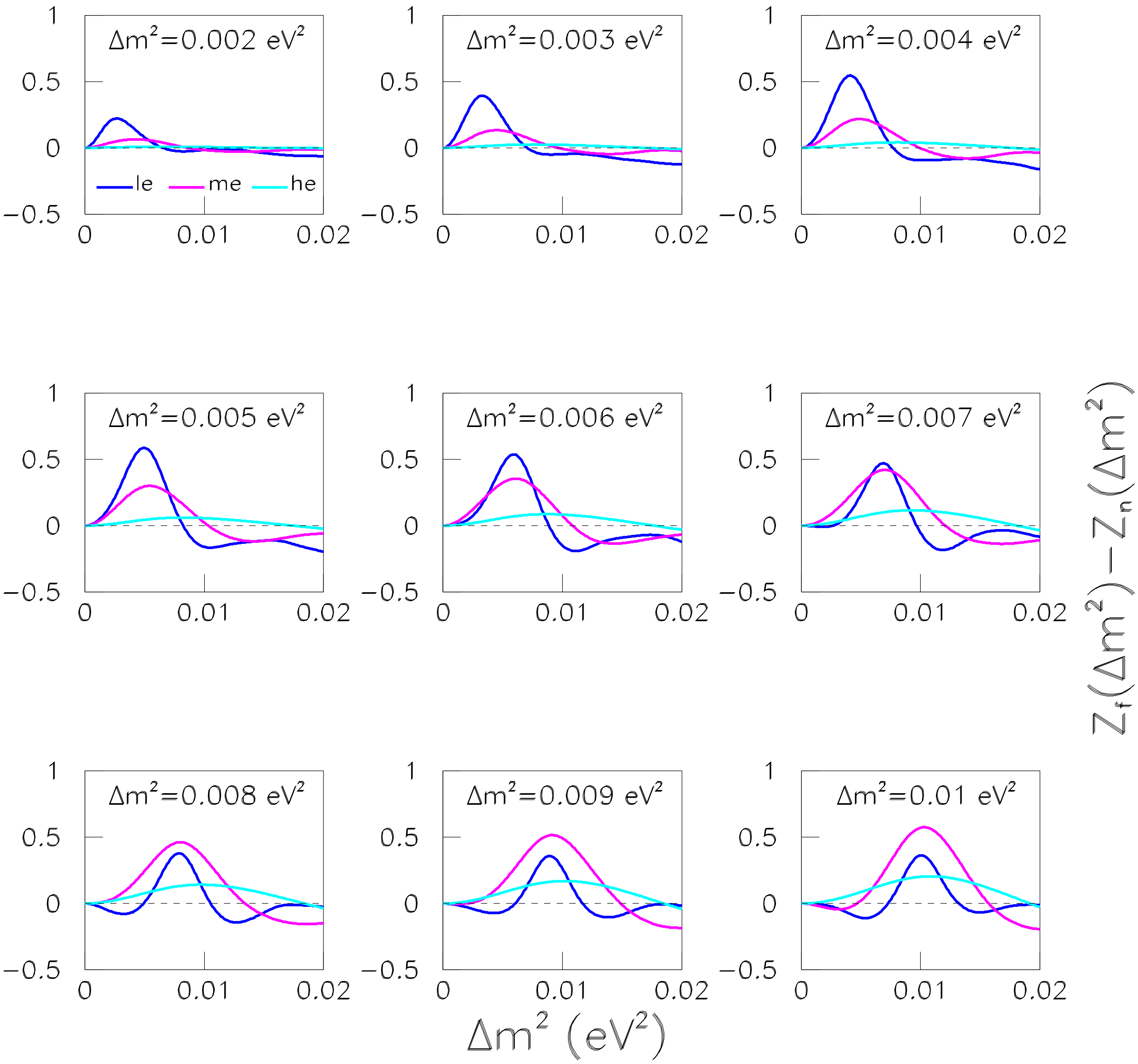


Figure 3: $\Delta Z(\Delta m^2)$ for several values of Δm^2 in each of the three beams.

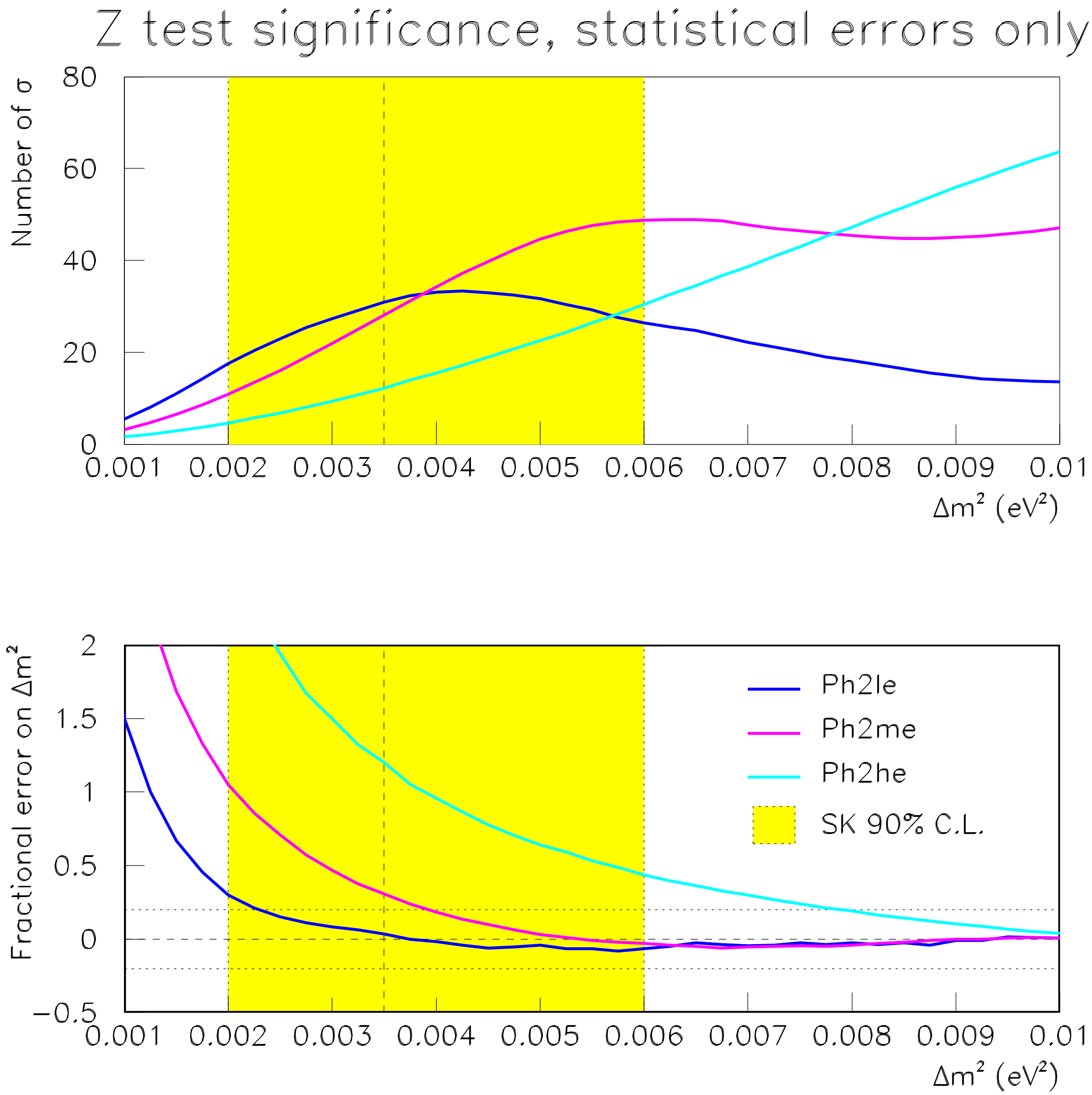


Figure 4: Top plot: the statistical significance of a signal in the Z-test for each of the three beams, assuming $\nu_\mu \to \nu_\tau$ with $\sin^2 2\theta = 1$ and a 10 kiloton year exposure of MINOS. Bottom plot: the fractional difference between the measured value of Δm^2, extracted from the Z-test signal, and the true value.

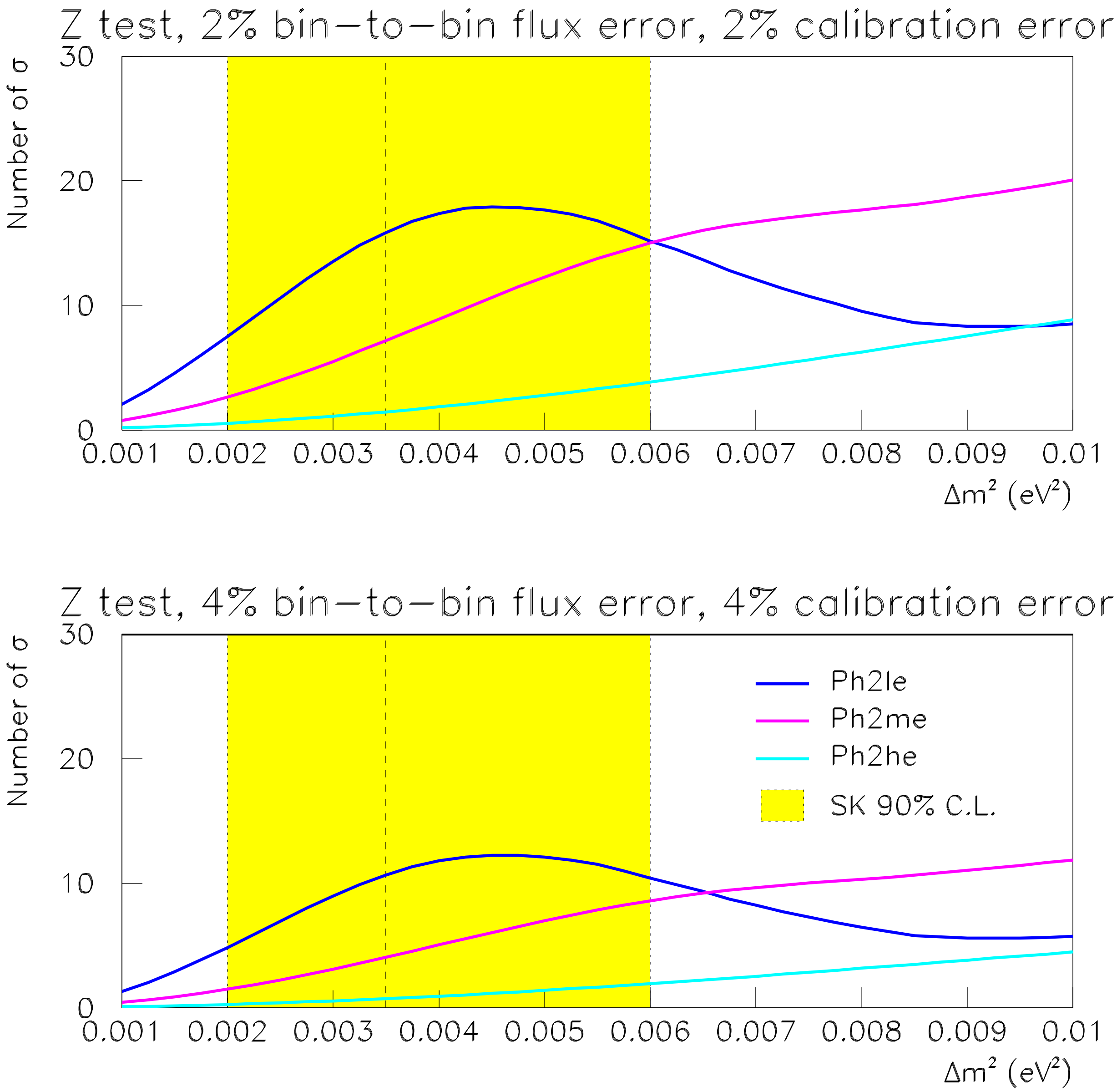


Figure 5: As the top plot of figure 4 but with systematic uncertainties applied.

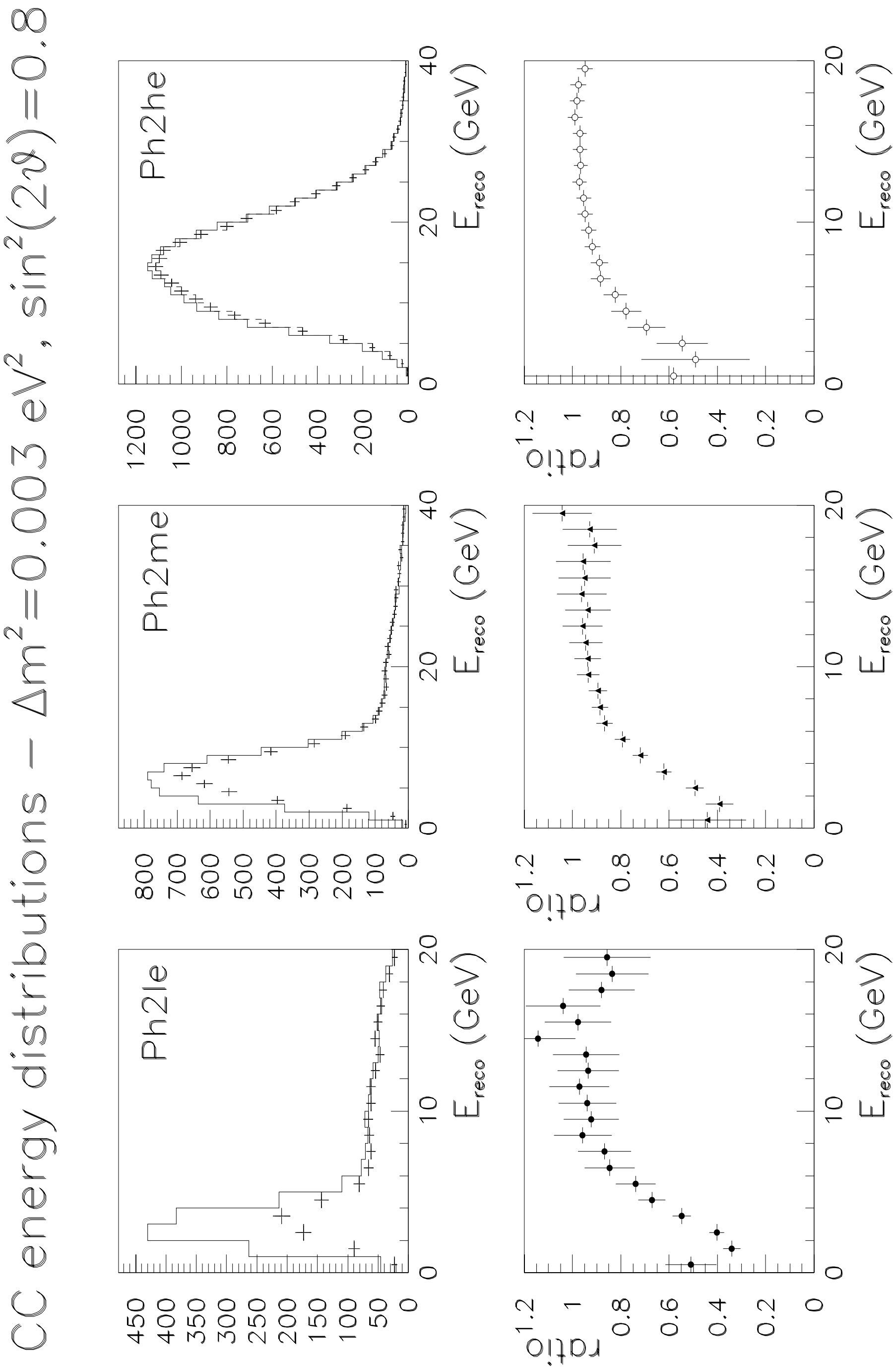


Figure 6: Top plots: energy distributions for a 10 kiloton year exposure in each of the three beams. Solid histograms: no oscillations, error bars: oscillations with $\Delta m^2 = 0.003$ eV^2 and $\sin^2 2\theta = 0.8$. Bottom plots: the ratio of the oscillated to the unoscillated histograms.

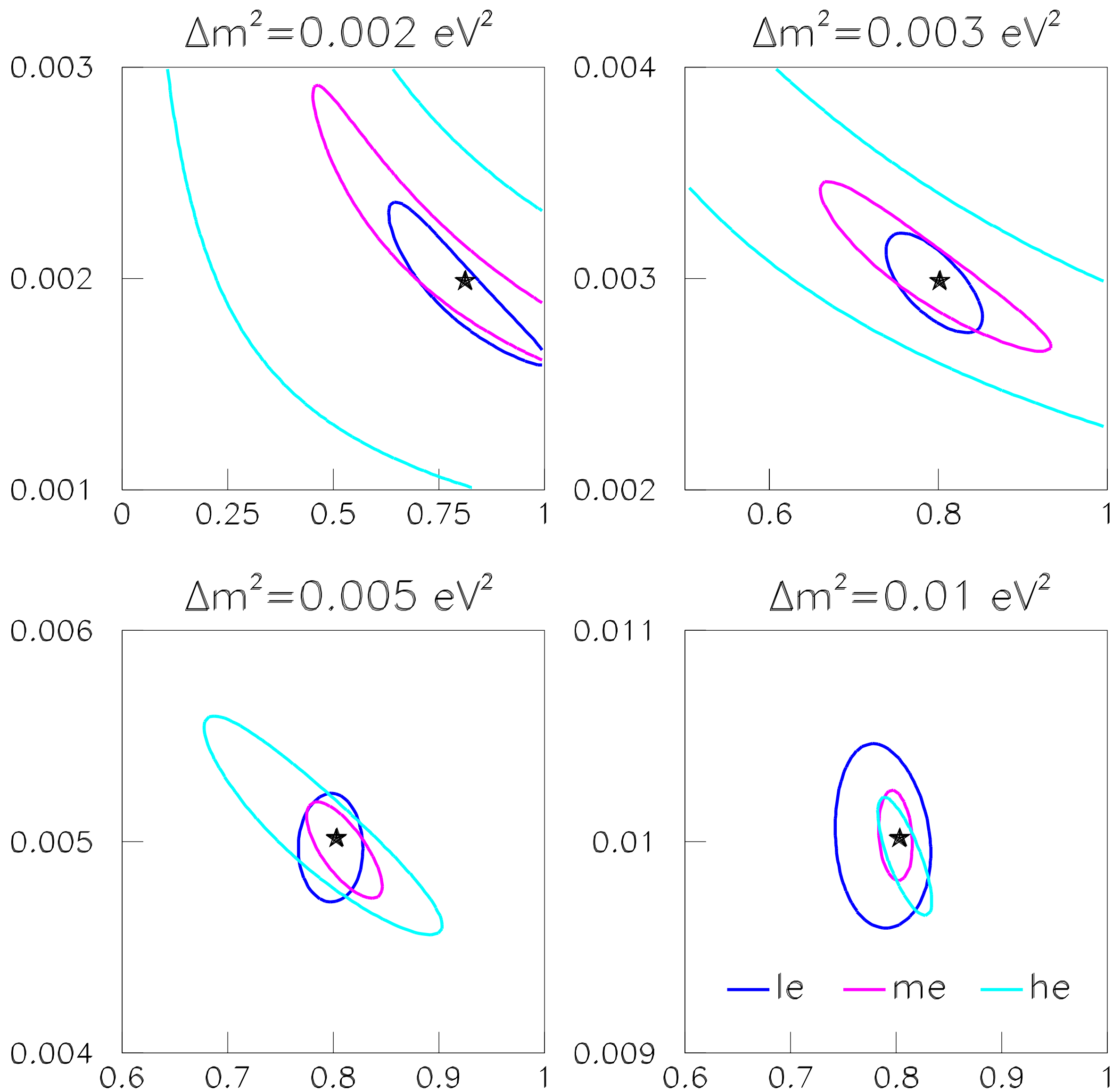


Figure 7: The 68% C.L. error contours in parameter space for four simulated experiments with $\sin^2 2\theta = 0.8$ and $\Delta m^2 = 0.002, 0.003, 0.005, 0.1$ eV^2.

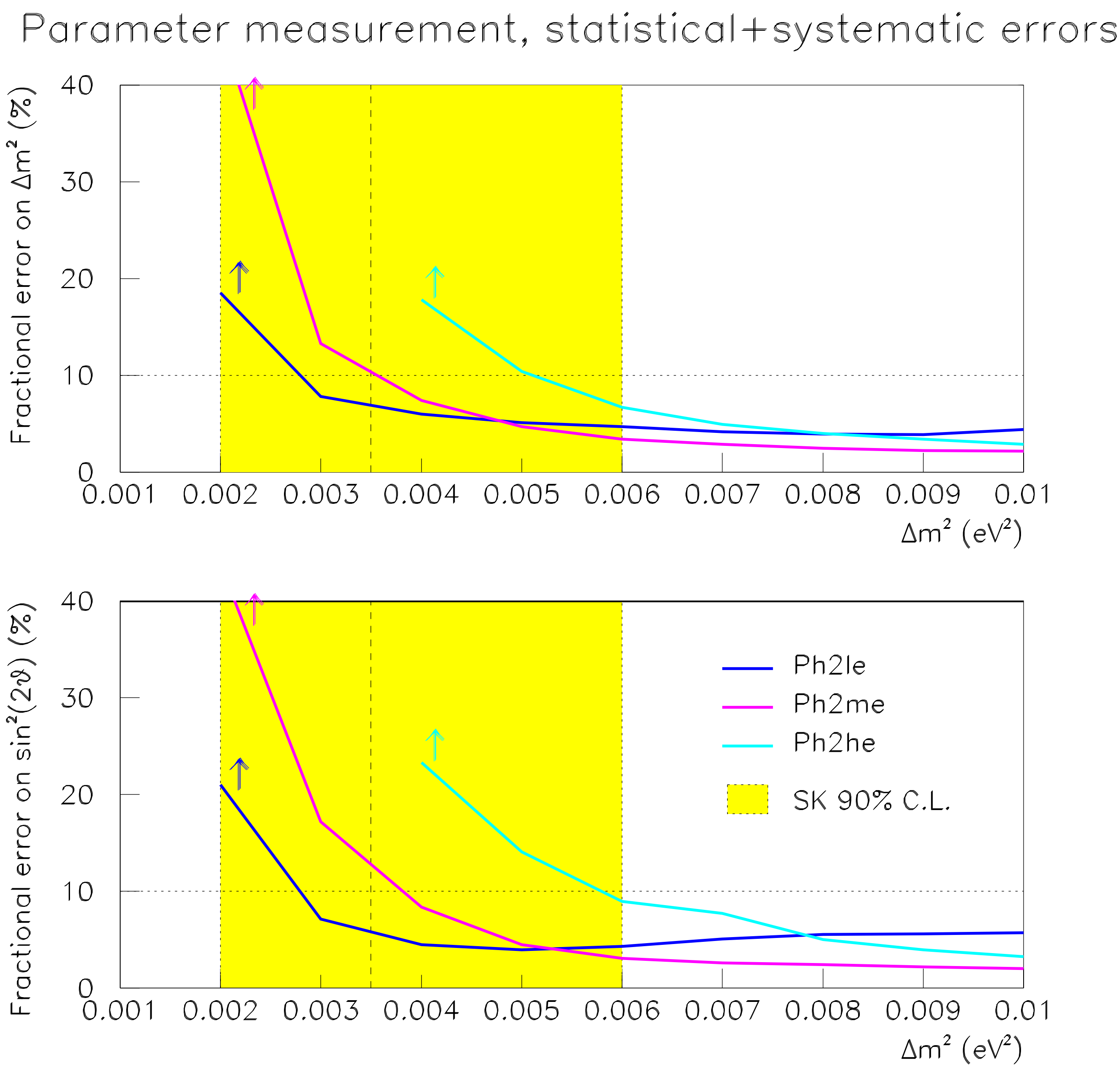
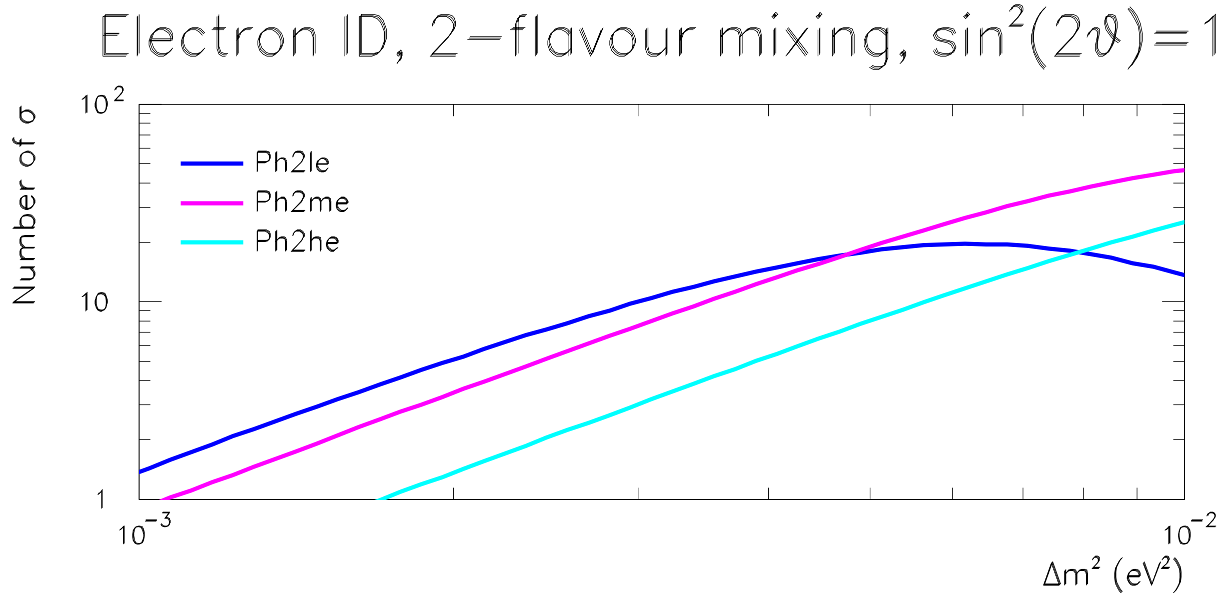


Figure 8: Parameter measurement errors on oscillation parameters Δm^2 and $\sin^2 2\theta$ for each of the three beams.



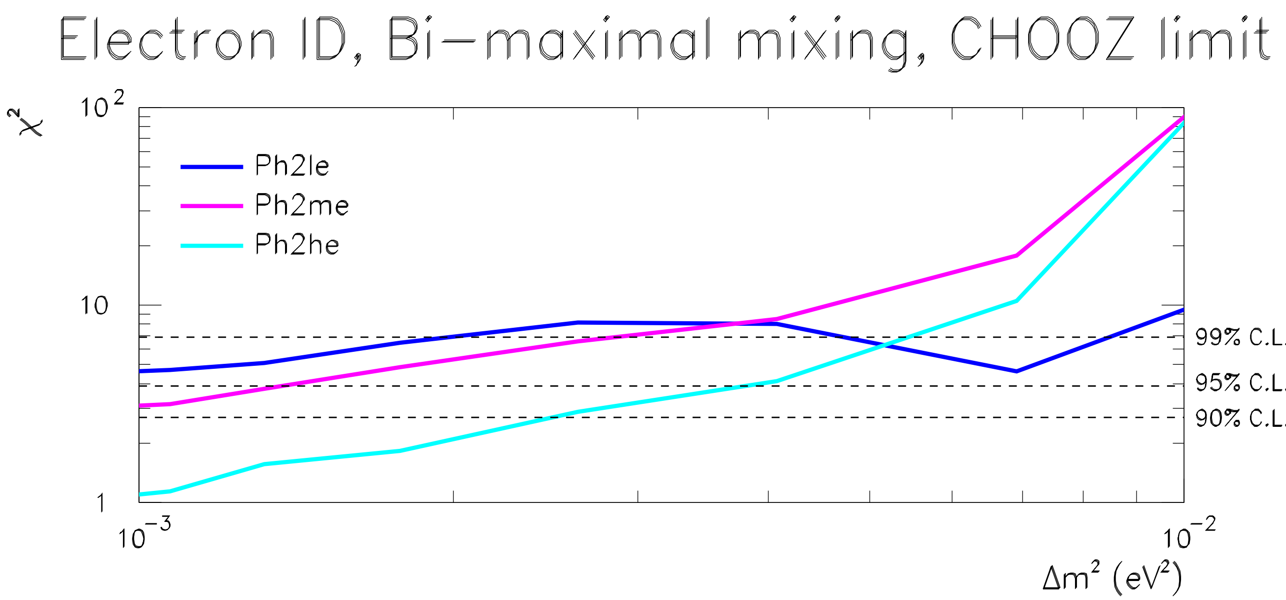


Figure 9: Top plot: the statistical significance of the electron appearance signal assuming $\nu_\mu \to \nu_e$ oscillations with $\sin^2 2\theta = 1$ and a 10 kiloton year exposure of MINOS. Bottom plot: the value of χ^2 obtained by comparing the numbers of electron-like events in the near and far detectors assuming three-flavour oscillations with large $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_e$ consistent with the CHOOZ limit.

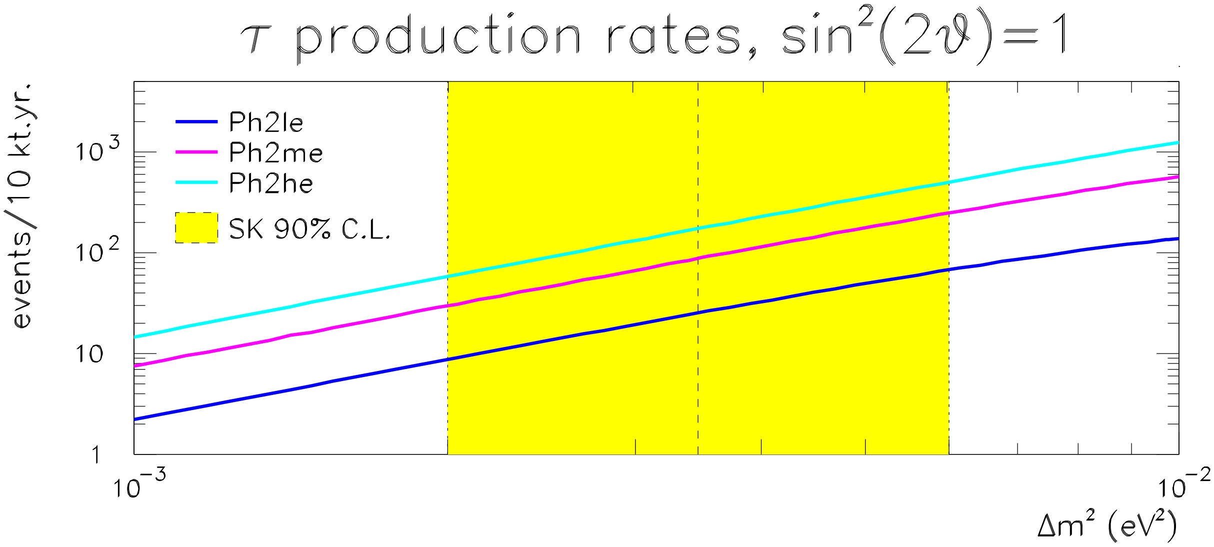


Figure 10: The ν_τ CC production rate in each of the three beams.

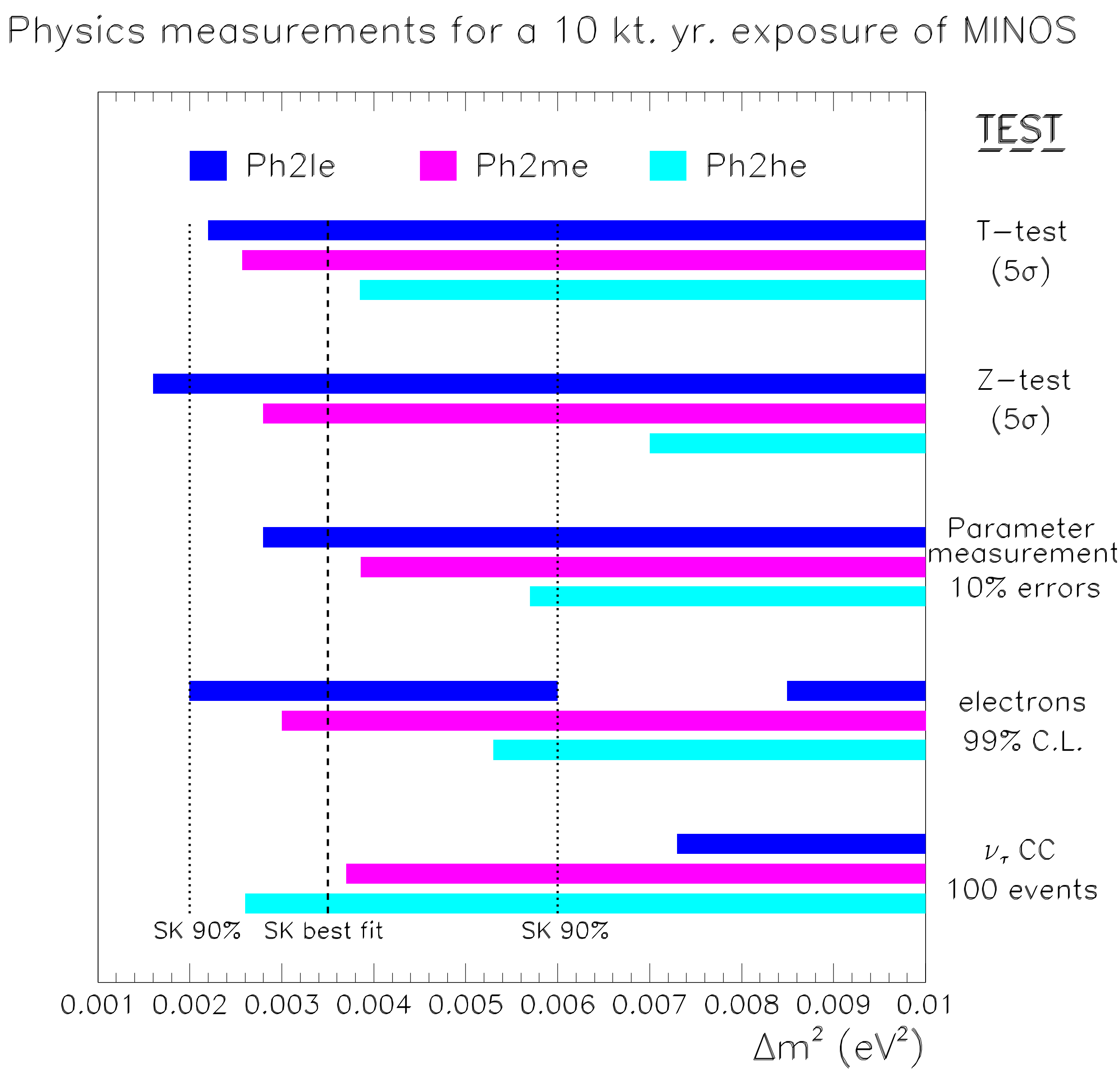


Figure 11: Summary of the physics measurements that are possible for a 10 kiloton year run in each of the three beams.